

**STUDIES ON
RELATIVISTIC STRINGS
AND
JET FRAGMENTATION**

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Lund, Sweden

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RELATIVISTIC STRINGS
AND
JET FRAGMENTATION

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Thesis for the degree of Doctor of Philosophy
(Avhandling för doktorsexamen)

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FÖRFATTARE
Söderberg, Bo Peter William

DOKUMENTTITEL OCH UNDERTITEL
Studies on relativistic strings and jet fragmentation

SAMMANFATTNING The first part of this thesis deals with the construction of action-angle variables for the massless relativistic string in one space-dimension. This is done in terms of the properties of the directrix. The mass squared is a simple linear combination of the action variables. The construction method is such, that different variables are obtained in different Lorentz frames. The variables obtained in the CM-frame and the light-cone frames have particularly appealing properties.

In the second part, a general iterative quark fragmentation model is constructed, that meets the requirements of Lorentz invariance, scaling, and quark-antiquark symmetry. This leads to a fragmentation function that apart from the scaling variable x , is allowed to depend on the mass and flavour of the meson.

NYCKELORDSTRING, ACTION-ANGLE VARIABLE, ONE-DIMENSIONAL; FRAGMENTATION MODEL, CASCADE

DOKUMENTTITEL OCH UNDERTITEL – SVENSK ÖVERSÄTTNING AV UTLÄNDSK ORIGINALTITEL
Relativistiska strängar och jet fragmentering

TILLÄMPNINGSOMRÅDE Första delen ger en klassisk-mekanisk analys av den relativistiska strängen, i en form som belyser kvantiseringsproblemet. Andra delen behandlar en fenomenologisk modell som kan användas för att underlätta jämförelser mellan teori och experiment som gäller stark växelverkan.

NYCKELORD KLASSISK MEKANIK, KVANTISERING, FRAGMENTERING

UTGIVNINGSDATUM år 1983 mån feb	ANTAL SID (inkl bilagor) 6	SPRÅK ☐ svenska ☒ engelska ☐ annat
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This thesis is based on the following papers

I. Hamilton-Jacobi Theory for the Massless Relativistic String

B. Andersson, G. Gustafson, B. Söderberg

Lund preprint LU TP 82-1

II. The Action-Angle Variables for the Massless Relativistic String in 1+1 Dimensions.

B. Söderberg, B. Andersson, G. Gustafson

Lund preprint LU TP 83-1

III. A General Model for Jet Fragmentation

B. Andersson, G. Gustafson, B. Söderberg

Lund preprint LU TP 83-2



"Det är skönare lyss till en sträng som brast
än att aldrig spänna en båge."

(Verner von Heidenstam)

1. Introduction

The strong interactions are commonly believed to be described by Quantum ChromoDynamics (QCD), a gauge field theory that was proposed in 1972. Although QCD has not been solved exactly, one can use perturbation theory to calculate cross sections in the lowest orders. The perturbation expansion is, however, valid only at large momentum transfers, corresponding to short distances. At large distances, QCD is believed to provide a constant attractive force that confines the quarks in a hadron to a limited space-time region. This constant force is thought to arise from the contraction of the field into a thin flux-tube (or string).

The Massless Relativistic String [1] was proposed in 1970 as a concretization of the Dual Resonance Model [2]. It has found an extensive use as a model for the quark confinement. In this model, a meson e.g. is thought of as a quark and an antiquark, tied together by a string. There is thus a strong motivation for investigating the properties, both classically and quantum mechanically, of the relativistic string.

2. The Hamilton-Jacobi Theory of the String

The first part of this thesis (papers I and II) concerns the Hamilton-Jacobi theory of the string in one space-dimension. More precisely, it deals with the problem of finding action-angle variables that describe the internal degrees of freedom of the system.

The solution to the equation of motion for the string is most easily expressed in terms of the directrix function. In one

space-dimension, the directrix has a very simple structure, and it can be parametrized in terms of a finite number of parameters.

In paper I we derive a very simple Poisson bracket (PB) algebra for the directrix parameters. The simple structure of the algebra makes possible the introduction of a reduction scheme, as is shown in paper II. The effect of this scheme is to simplify the directrix, while "splitting off" one pair of canonical variables. When iterated, it leads to a complete set of canonical variables, that in a natural way are related to the action-angle variables of the string.

The properties of the action-angle variables are investigated, and it is shown that they have especially nice features when evaluated in the CM frame, or in one of the light-cone frames.

3. Quark Fragmentation

In experiments, where hadrons are involved in a collision, the final state often contains clusters (jets) of particles going out in more or less the same direction. Phenomenologically, this can be explained as follows: When one of the quarks (or antiquarks) inside a hadron is given a large momentum in a collision, it tries to escape from its fellow partons. In its wake a flux-tube is stretched out. When the flux-tube (or string) reaches a certain length, it becomes energetically favourable for it to break into two sub-strings by the materialization of a new quark-antiquark pair. The substrings will continue to be stretched, and break into more sub-strings. Eventually, the breaking will stop, and we will have a final state consisting of a number of stable string fragments, that can be associated with the particles in a jet. The quark (or antiquark) is said to fragment into hadrons.

Several models for the fragmentation of a quark have been proposed, one of the most successful having been developed here in

Lund [3]. However, they all suffer from either the lack of Lorentz invariance, or a quark-antiquark asymmetry, or the impossibility of using physical masses for the mesons.

The second part of this thesis (paper III) treats the problem of finding an iterative fragmentation model without any of these inconsistencies. An iterative fragmentation model is generically defined in the following way: By applying a large boost in the string direction, one can always consider the mesons as split off one by one from the end of the string. For every meson we define the variable x as the fraction of the remaining light-cone momentum taken by the meson. The variable x is assumed to be statistically distributed between 0 and 1 according to a fragmentation function $f(x)$.

In paper III we show that it is possible to construct an iterative fragmentation model that meets the following requirements:

- i) Lorentz invariance
- ii) Symmetry between quarks and antiquarks.
- iii) Independent fragmentation of sub-strings, which in the light-cone frame leads to a fragmentation function that is allowed to depend on the flavour and mass of the meson, but is otherwise universal (as long as the mass of the remaining string is large).

The general solution to these requirements is given by the fragmentation function

$$f(x)dx \propto \frac{dx}{x} x^{a_1} \left(\frac{1-x}{x}\right)^{a_2} e^{-\frac{bm^2}{x}}$$

where m is the mass and b is a universal parameter, while a_1 and a_2 are parameters that may depend on the meson flavour and antiflavour, respectively.

This model is further investigated in paper III, and is shown to have several appealing features.

Acknowledgements

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